

Correction du Devoir Surveillé N°1 S^o2 1BAC SE

Exercice 1

$$P(x) = \sin(2x) - \cos(2x) + 1 + \sin x + \cos x$$

1) a) $\sin(2x) - \cos(2x) + 1 = 2\sin x \cos x - (1 - 2\sin^2(x)) + 1$
 $= 2\sin x \cos x - 1 + 2\sin^2(x) + 1$

b) $\sin(2x) - \cos(2x) + 1 = 2\sin x (\cos x + \sin x)$

c) on a $\sin x + \cos x = r \cos(x - \alpha)$

avec $r = \sqrt{1^2 + 1^2} = \sqrt{2}$ et $\cos \alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ et $\sin \alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
 donc $\alpha = \frac{\pi}{4}$

d'où $\sin x + \cos x = \sqrt{2} \cos(x - \frac{\pi}{4})$

2) a) $P(x) = \sin(2x) - \cos(2x) + 1 + \sin x + \cos x$
 $= 2\sin x (\cos x + \sin x) + (\cos x + \sin x)$
 $= (2\sin x + 1)(\cos x + \sin x) = \sqrt{2}(2\sin x + 1) \cos(x - \frac{\pi}{4})$

3) a) Résolvons dans \mathbb{R} $P(x) = 0$
 $2\sin x + 1 = 0$ ou $\cos(x - \frac{\pi}{4}) = 0$
 $\sin x = -\frac{1}{2}$ ou $\cos(x - \frac{\pi}{4}) = 0$
 $x = -\frac{\pi}{6} + 2k\pi$ ou $x = -\frac{5\pi}{6} + 2k\pi$ ou $x - \frac{\pi}{4} = \frac{\pi}{2} + k\pi$ / $k \in \mathbb{Z}$

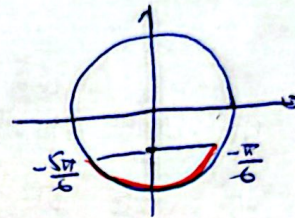
$S = \{ -\frac{\pi}{6} + 2k\pi; -\frac{5\pi}{6} + 2k\pi; \frac{3\pi}{4} + k\pi / k \in \mathbb{Z} \}$

b) \sin sur $]-\pi, \pi]$

$x = -\frac{\pi}{6}$; $x = -\frac{5\pi}{6}$; $x = -\frac{\pi}{4}$; $x = \frac{3\pi}{4}$

$S = \{ -\frac{5\pi}{6}; -\frac{\pi}{2}; -\frac{\pi}{4}; \frac{3\pi}{4} \}$

4) on pose $2\sin x + 1 \leq 0$
 a) $\sin x \leq -\frac{1}{2}$

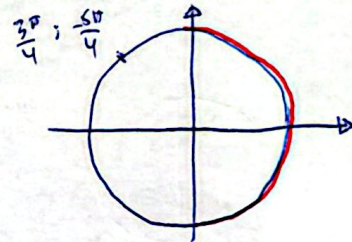


x	$-\pi$	$-\frac{5\pi}{6}$	$-\frac{\pi}{6}$	π
$2\sin x + 1$		+	-	+

b) on pose $\cos(x - \frac{\pi}{4}) \geq 0$

on pose $X = x - \frac{\pi}{4}$ avec $x \in]-\pi, \pi]$

donc $\cos X \geq 0$ $X \in]-\frac{5\pi}{4}; \frac{3\pi}{4}]$



$$\begin{aligned} -\frac{5\pi}{4} &\leq X \leq \frac{3\pi}{4} \\ -\frac{5\pi}{4} + \frac{\pi}{4} &\leq x - \frac{\pi}{4} \leq \frac{3\pi}{4} + \frac{\pi}{4} \\ -\frac{4\pi}{4} &\leq x \leq \frac{4\pi}{4} \end{aligned}$$

x	$-\pi$	$-\frac{\pi}{4}$	$\frac{3\pi}{4}$	π
$\cos(x - \frac{\pi}{4})$		-	+	-

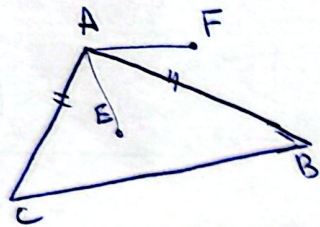
c) Tableau de signe de $P(x)$

x	$-\pi$	$-\frac{5\pi}{6}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$\frac{3\pi}{4}$	π
$2\sin x + 1$		+	-	-	+	+
$\cos(x - \frac{\pi}{4})$		-	-	+	+	-
$P(x)$		-	+	-	+	-

$S = [-\frac{5\pi}{6}; -\frac{\pi}{4}] \cup [-\frac{\pi}{6}; \frac{3\pi}{4}]$

exercice 2

v)



- 1) $r(C) = B$ car $AC = AB$ et $(\vec{AC}, \vec{AB}) \equiv \frac{\pi}{2} [2\pi]$
- 3) on a $r(C) = B$ et $r(E) = F$ donc $CE = BF$
- 4) on a $r(C) = B$ et $r(E) = F$
donc $(\vec{CE}, \vec{BF}) \equiv \frac{\pi}{2} [2\pi]$
donc $(CE) \perp (BF)$